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Cost-sharing eco-compensation for transboundary pollution control: a differential game under alternative leadership structures

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Transboundary pollution creates a governance problem in which the region controlling emissions does not fully bear the resulting environmental damage, while the affected region suffers pollution exposure without direct control over source-side decisions. This study examines whether cost-sharing eco-compensation can mitigate this incentive mismatch under decentralized environmental governance. A continuous-time differential game is developed between a pollution source region and an affected region, incorporating emission-generating production, cleanup effort, pollution accumulation, and cross-border diffusion. The analysis compares a decentralized benchmark with four cost-sharing mechanisms: source-led source-side cleanup cost sharing, affected-region-led source-side cleanup subsidy, government-led polluter-pays compensation, and government-led beneficiary-pays cost sharing. The results show that cost-sharing eco-compensation improves environmental outcomes mainly through the cleanup-cost channel rather than by directly disciplining emissions. Leadership structure is central to contract design. Source-region leadership pushes the sharing ratio toward the affected region's participation boundary, whereas affected-region leadership generates a more conservative ratio because the affected region internalizes its own subsidy cost. Government leadership selects the ratio according to aggregate welfare and can support mechanisms that are difficult to sustain through local bargaining alone. Numerical simulations further show that the effectiveness of cost sharing depends on the direction and intensity of pollution spillovers, the feasibility of participation constraints, and the relative efficiency of source-side versus local cleanup. These findings clarify both the potential and the limits of cost-sharing eco-compensation as a second-best instrument for decentralized transboundary pollution control.

KEYWORDS
cost-sharing contract, decentralized environmental governance, eco-compensation, leadership structure, Stackelberg differential game, transboundary pollution

1 Introduction

Who should pay for pollution that crosses borders? This question is simple in form but difficult in practice. Pollutants released in one jurisdiction may travel through rivers, watersheds, airsheds, and ecological corridors, affecting regions that do not control the original source of emissions. According to UN-Water (2024), transboundary rivers account for about 60% of global freshwater flows, and 153 UN Member States rely on waters that flow from or to another country. Air pollution follows a similar spatial logic: emissions generated in one place can alter environmental quality elsewhere (Jørgensen and Zaccour, 2001a). These facts reveal a central tension in environmental governance. Pollution-control decisions are often made locally, but their environmental consequences are not. The source region controls emission-generating production and source-side cleanup, yet it does not fully bear the damage transmitted to neighboring regions. The affected region suffers from cross-border pollution exposure, but it cannot directly determine the source region's emissions or cleanup choices (Fang et al., 2026). This separation between control and exposure weakens decentralized pollution-control incentives. Wang et al. (2020) link weak border regulation in China to local officials' promotion incentives, while Sulaymon et al. (2021) show that regional air-pollution transport can significantly affect local air quality. Transboundary pollution is therefore not only a physical spillover problem. It is also an institutional incentive problem.

A standard policy response is to price emissions directly. Emission taxes raise the marginal cost of polluting activities, while tradable permit systems impose an emissions cap and allow regulated agents to trade allowances. These instruments are attractive because they target the emission decision itself. The World Bank (2026) and Yu et al. (2025) reports that direct carbon pricing covers nearly 30% of global greenhouse gas emissions, and ICAP reports that 41 emissions trading systems are currently in force worldwide. Yet transboundary governance often lacks the institutional conditions required for these instruments to work effectively. A cross-regional emission tax requires a clear taxing authority, reliable emission monitoring, and enforceable rules across jurisdictions. A permit system requires a common cap, allowance allocation, trading infrastructure, and credible enforcement. When environmental damage crosses borders but regulatory authority remains fragmented, the most direct instrument in theory may not be the most feasible instrument in practice.

Eco-compensation and cost-sharing arrangements offer a different policy logic. They do not directly price emissions. Instead, they reallocate the cost of verifiable cleanup activities across jurisdictions. One region may pay part of another region's pollution-control cost, thereby changing the effective marginal cost of cleanup and inducing additional abatement effort. Recent studies show that ecological compensation can improve cross-regional environmental governance. Ding et al. (2022) find that cost sharing can increase ecological protection effort in basin governance. Xu et al. (2024) show that the direction of compensation affects pollution-control effort and welfare in a transboundary watershed. Yi et al. (2024) extend ecological compensation to a multi-region watershed setting. Chen et al. (2025) and Wang et al. (2025) further show that ecological compensation can help internalize cross-border water-pollution externalities. Related research on payments for ecosystem services also shows that payment design matters, because payments and penalties shape program performance (Kim et al., 2024). However, existing research has not fully explained how cost-sharing eco-compensation should be designed when regions differ in control authority, damage exposure, and bargaining position. Three design choices remain especially important. First, cost sharing operates through cleanup costs rather than emissions. Second, the shared cost may be source-side cleanup or affected-region local remediation. Third, the cost-sharing ratio may be chosen by the source region, the affected region, or an upper-level government. These choices determine whether eco-compensation functions as an incentive mechanism or merely as a fiscal transfer.

The central policy issue is therefore not whether cost sharing can be mathematically introduced into a pollution-control model, but when it can serve as a feasible governance instrument under fragmented authority. In many transboundary pollution settings, direct emission pricing is institutionally demanding because it requires unified monitoring, taxation or permit allocation, and cross-jurisdictional enforcement. By contrast, cleanup projects, remediation expenditures, and pollution-control works are more likely to be verified and written into interregional agreements. This makes cost-sharing eco-compensation a practical and policy-relevant instrument: it does not replace emission taxes or tradable permits, but it may improve decentralized governance when direct emission control is difficult to implement.

This paper therefore treats the cost-sharing ratio not merely as a mathematical parameter, but as an institutional design variable. Different leaders face different objectives and constraints. A source region may use its control over source-side cleanup to shift part of the cost to the affected region; an affected region may voluntarily subsidize source-side cleanup to reduce cross-border damage; and an upper-level government may impose a cost-sharing rule from the perspective of aggregate welfare. These differences determine whether eco-compensation functions as an incentive-compatible governance arrangement or degenerates into a fiscal transfer with limited environmental effect.

These unresolved design choices raise three research questions.

- RQ1: How is the cost-sharing ratio determined when contract leadership is held by the source region, the affected region, or an upper-level government?
- RQ2: Does cost-sharing eco-compensation affect environmental behavior through emissions, source-side cleanup, or affected-region cleanup?
- RQ3: Under what externality and feasibility conditions can cost sharing improve decentralized transboundary pollution control?

To answer these questions, this study develops a continuous-time differential game between a pollution source region and an affected region. Pollution accumulates over time and diffuses across borders. Each region chooses emission-generating production and cleanup effort. The decentralized benchmark captures the outcome when each region maximizes its own welfare and does not fully internalize cross-border environmental damage. The analysis then compares four cost-sharing mechanisms: source-led source-side cleanup cost sharing, affected-region-led source-side cleanup subsidy, government-led polluter-pays compensation, and government-led beneficiary-pays cost sharing. This framework separates three elements that are often bundled together in policy discussions: the object of cost sharing, the party that leads contract design, and the incentive channel through which the contract affects pollution control.

The results show that cost-sharing eco-compensation operates mainly through the cleanup-cost channel rather than the emission channel. Source-side cost sharing reduces the source region's effective cleanup cost and increases source-side treatment. Polluter-pays compensation reduces the affected region's local cleanup cost and increases local remediation. Leadership structure also matters. Source-region leadership pushes the sharing ratio toward the affected region's participation boundary. Affected-region leadership produces a more conservative ratio because the affected region internalizes its own subsidy burden. Government leadership selects the ratio from the perspective of aggregate welfare. Numerical simulations further show that cost-sharing performance depends on the direction and intensity of pollution spillovers, the feasibility of participation constraints, and the relative efficiency of source-side versus local cleanup.

This study makes three policy-oriented contributions. First, it reframes cost-sharing eco-compensation as an incentive-design problem under decentralized transboundary governance. The key question is not simply who transfers money to whom, but whether the transfer changes the effective marginal cost of verifiable cleanup effort. Second, it distinguishes two operational objects of cost sharing: source-side cleanup and affected-region local remediation. These two objects correspond to different incentive channels, different welfare effects, and different conditions of applicability. Third, it endogenizes the cost-sharing ratio under alternative leadership structures. By comparing source-region leadership, affected-region leadership, and government leadership, the paper shows how bargaining power, participation constraints, and aggregate-welfare considerations shape the design and feasibility of eco-compensation mechanisms.

2 Literature review

Transboundary pollution and its impact on environmental governance have been extensively studied. Examples of the topic include pollution transmission networks (Xue and Wang, 2024), myopia and foresight (Benchekroun and Martín-Herrán, 2016), ecological compensation (Jiang et al., 2019; Ding et al., 2022; Xing, 2023; Xu et al., 2024; Yi et al., 2024), cooperative strategies (Huang et al., 2016; Jiao et al., 2021; Yu et al., 2024), incentives (Fan et al., 2023) and transfer payment (Kim et al., 2024; Tan et al., 2023; Izquierdo-Tort et al., 2024). The literature relevant to this study can be organized into three streams: (1) transboundary pollution and decentralized environmental governance, (2) dynamic games of transboundary pollution control, and (3) ecological compensation, cost sharing, and environmental contract design.

2.1 Transboundary pollution and decentralized environmental governance

Transboundary pollution reflects a mismatch between pollution-control authority and environmental damage exposure. The source region controls emissions and cleanup efforts, while part of the damage is transmitted to neighboring jurisdictions. Empirical studies show that decentralized governance may generate free-riding incentives when pollution-control benefits spill across borders. Sigman (2005) documents interjurisdictional free riding in water pollution control. Kahn et al. (2015) show that pollution can concentrate near political borders. Sulaymon et al. (2021) further show that regional air-pollution transport affects local air quality, confirming the physical basis of cross-regional dependence.

Recent studies increasingly evaluate ecological compensation as a coordination tool. Wang et al. (2025) show that ecological compensation can help internalize transboundary pollution externalities. Shan et al. (2026) extend horizontal ecological compensation to transboundary marine governance, and Yang (2026) links watershed ecological compensation to pollution abatement and green efficiency. However, these studies mainly examine whether transboundary pollution or ecological compensation affects environmental outcomes, while paying less attention to how an eco-compensation arrangement is endogenously designed when jurisdictions differ in control authority, damage exposure, and bargaining position.

2.2 Dynamic games of transboundary pollution control

Dynamic game models are widely used because emissions and cleanup efforts affect pollution stocks over time. Jørgensen and Zaccour (2001b) establish the cooperative versus non-cooperative logic of pollution control. Bertinelli et al. (2014) introduce carbon capture and storage into a transboundary differential game. Benchekroun and Martín-Herrán (2016) show that foresight changes strategic abatement. de Frutos et al. (2019) highlight spatial effects in multiregional pollution games, and Boucekkine et al. (2021) connect firm-level and aggregate pollution control.

Recent work incorporates richer institutional structures. Xue and Wang (2024) examine pollution transmission networks. Yuan et al. (2024) model trans-jurisdictional river-basin pollution management with multiple stakeholders. Yi et al. (2024) and Xu et al. (2024) introduce ecological compensation into multi-region or upstream-downstream watershed games. Lu (2025) and Yang et al. (2025) further consider Stackelberg or central-government-led pollution-control settings. These studies show that dynamic interaction, stakeholder heterogeneity, and compensation design affect pollution-control performance. Although dynamic pollution games have clarified the intertemporal inefficiency of decentralized environmental governance, less is known about how a cost-sharing contract alters equilibrium behavior through cleanup-cost incentives rather than direct emission control.

2.3 Ecological compensation, cost sharing, and environmental contract design

Ecological compensation and PES studies provide the contract-design logic for this paper. Jiang et al. (2019) introduce ecological compensation into a stochastic differential game. Ding et al. (2022) show that cost sharing increases ecological protection efforts in cross-regional basin governance. Huang (2023) finds that compensation does not always improve both environmental quality and welfare. Xu et al. (2024) compare no compensation, one-way compensation, and two-way compensation, showing that compensation direction affects pollution-control effort and welfare.

The PES and environmental contract literature show that compensation should be treated as an incentive contract rather

TABLE 1 Related literature.

Authors	Dynamic pollution stock	Eco-compensation/Cost sharing	Endogenous cost-sharing ratio	Leadership structure
Sulaymon et al. (2021)	No	No	No	Decentralized governance
Ding et al. (2022)	Yes	Yes	No	Cooperation and cost-sharing mechanisms
Xue and Wang (2024)	Yes	No	No	Networked or multi-stakeholder governance
Wang et al. (2025)	No	Yes	No	Ecological compensation policy evaluation
Nazari et al. (2026)	No	Yes	No	PES scheme design
Present paper	Yes	Yes	Yes	Source-led, affected-region-led, and government-led mechanisms

TABLE 2 Baseline parameter values and interpretations.

Parameter	Value	Interpretation
ρ	0.05	Annual discount rate
α_i	1.25	Production-benefit scale of the source region
α_j	0.85	Production-benefit scale of the affected region
φ_i	0.18	Pollution transfer from the source region to the affected region
φ_j	0.14	Reverse pollution transfer
ϕ_i	0.26	Natural dissipation rate in the source region
ϕ_j	0.30	Natural dissipation rate in the affected region
η_i	0.060	Marginal pollution damage in the source region
η_j	0.220	Marginal pollution damage in the affected region
l_i	4.50	Treatment efficiency in the source region
l_j	4.50	Treatment efficiency in the affected region
c_i	2.10	Source-side treatment cost coefficient
c_j	4.50	Affected-region local treatment cost coefficient
$x_i(0)$	26.00	Initial pollution stock in the source region
$x_j(0)$	22.00	Initial pollution stock in the affected region

than a simple transfer. Kim et al. (2024) analyze payments and penalties, while Izquierdo-Tort et al. (2024) show that contract redesign improves cost-effectiveness. Schaub et al. (2025), Süring and Lundhede (2025), Nazari et al. (2026), and Grahl et al. (2026) further emphasize result-based payments, compensation requirements, PES sustainability, and collective incentive design. Existing studies show that compensation can improve environmental governance, but they generally do not distinguish whether the shared cost is source-side cleanup or affected-region cleanup, nor do they systematically examine how different institutional leaders select the cost-sharing ratio.

Table 1 shows that the closest studies examine dynamic ecological compensation and cost-sharing scenarios. The distinction of this paper is more specific: it treats leadership over the cost-sharing ratio as the institutional variable of interest and compares how source-led, affected-region-led, and government-led mechanisms affect cleanup incentives and welfare outcomes.

3 Model formulation

The model considers a continuous-time differential game of transboundary pollution between two neighboring regions, a source region and an affected region. At each point in time, the regions choose emission-generating production intensity and abatement effort. These choices determine contemporaneous economic returns and control costs, while also governing the law of motion of pollution stocks. Under decentralized decision-making, each region maximizes its own discounted payoff and does not internalize the full cross-border environmental damage generated by its actions. The benchmark setting thus captures the equilibrium consequences of strategic regional behavior in the absence of any coordinating arrangement and serves as the reference point for the compensation regimes analyzed below.

3.1 Governance asymmetry between the two regions

A central feature of transboundary pollution is the asymmetry between the distribution of environmental damage and the distribution of decision-making power. The source region directly controls the key source-side decisions, namely pollution-generating production and abatement effort, and therefore has substantial influence over current emissions and the future path of pollution accumulation. However, because part of the environmental damage is transmitted across regional borders, the source region does not fully internalize the losses imposed on the neighboring region. By contrast, the affected region bears a share of the environmental consequences generated by cross-border pollution, yet under decentralized governance, it cannot directly determine the source region's production or abatement choices. In other words, the region that can effectively alter the pollution path does not bear the full marginal damage, whereas the region that suffers from that damage lacks direct control over the underlying decisions. This mismatch between control and exposure constitutes the core governance asymmetry in transboundary pollution and provides the main rationale for introducing an interregional incentive mechanism.

3.2 Pollution stock dynamics

Let $x_h(t)$ denote the pollution stock in region $h \in \{i, j\}$ at time t. The evolution of pollution stocks follows the coupled differential equations adopted from the transboundary pollution game literature (Bertinelli et al., 2014; Xue and Wang, 2024): The dynamic state of the pollution stock is

$$\begin{cases} \dot{x}_i(t) = e_i(t) - l_i u_i(t) + \varphi_j x_j(t) - \left(\phi_i + \varphi_i\right) x_i(t) \\ \dot{x}_j(t) = e_j(t) - l_j u_j(t) + \varphi_i x_i(t) - \left(\phi_j + \varphi_j\right) x_j(t) \\ x_i(0) = x_i^0 \geq 0; x_j(0) = x_j^0 \geq 0 \end{cases} \tag{1}$$

where $l_h > 0$ measures the effectiveness of abatement effort in reducing the local pollution stock (El Ouardighi et al., 2016). The parameter $\phi_h > 0$ denotes the natural self-purification rate of pollution in the region h (Xue and Wang, 2024), while $\varphi_h \in [0, 1]$ captures the intensity with which pollution diffuses from region h to the neighboring region (de Frutos et al., 2019). This specification, standard in the differential game literature on transboundary pollution (Boucekkine et al., 2021), implies that the current pollution stock depends on current emissions and abatement, the inherited stock, and cross-border spillovers.

3.3 Instantaneous regional welfare

Following the environmental economics literature, each region's instantaneous welfare comprises three components: economic returns from production, abatement costs, and environmental damages.

3.3.1 Economic returns

Since the major pollutant emissions of region h mainly arise from the production of industrial products, the emission level $e_h(t)$ of region h over the planning horizon is directly related to its production level $q_h(t)$. The emission level of region h can be expressed in linear form as $e_h(t) = \omega_h q_h(t)$, where $\omega_h > 0$ is the marginal coefficient of $e_h(t)$ with respect to $q_h(t)$. Following Breton et al. (2006), the benefit function of region h can be derived from the output of the major pollutant as

$$R_h(t) = \ln\left(\alpha_h q_h(t)\right) = \ln\left(\alpha_h e_h(t)\right) - \ln \omega_h \tag{2}$$

Here, the coefficient a_h is a constant. Since the marginal coefficient ω_h is constant, let $b_h = -\ln\,[MediumSpace]\omega_h$ denote the constant term in Equation 3. Then the revenue function of region h at time can be simplified as

$$R_h(t) = \ln\left(\alpha_h e_h(t)\right) + b_h \tag{3}$$

3.3.2 Abatement costs

Following the pollution control literature (Jiang et al., 2019; Xue and Wang, 2024), abatement costs are convex and increasing in effort:

$$C_h(u_h(t)) = \frac{c_h}{2} u_h(t)^2 \tag{4}$$

where $c_h > 0$ is the cost parameter. This quadratic specification reflects the empirically observed property that marginal abatement costs increase with effort (Lichtenberg, 2021; Kim et al., 2024).

3.3.3 Environmental damage

Damage from accumulated pollution is specified as linear in the stock

$$D_h(x_h(t)) = \eta_h x_h(t) \tag{5}$$

where $\eta_h > 0$ measures the marginal welfare loss per unit of pollution stock. This linear damage specification is widely used in dynamic transboundary pollution games for analytical tractability (Benchekroun and Martín-Herrán, 2016; Boucekkine et al., 2021).

3.3.4 Instantaneous payoff

Combining the three components, region h's instantaneous net payoff is:

$$\pi_h(t) = b_h + \ln\left(\alpha_h e_h(t)\right) - \frac{c_h}{2} u_h(t)^2 - \eta_h x_h(t) \tag{6}$$

This formulation reflects the standard intertemporal trade-off in dynamic pollution control (Jørgensen and Zaccour, 2001a; Smala Fanokoa et al., 2011): higher current production generates immediate economic gains but contributes to future environmental burdens, while higher abatement effort reduces future damages at the cost of current expenditure.

In practice, interregional compensation schemes are more likely to be designed around verifiable abatement expenditures than around source-region emissions, which are more difficult to contract upon because they are closely tied to local production decisions. Eco-compensation is therefore specified as cost sharing over verifiable cleanup expenditures; depending on the mechanism, the shared cost may be source-side cleanup cost or affected-region local cleanup cost. The contract reduces the effective marginal cleanup cost of the region whose cleanup expenditure is shared, rather than directly regulating the polluting region's emissions.

4 Benchmark model

This section establishes the decentralized benchmark. Each region independently chooses its emission intensity and abatement effort to maximize its own discounted payoff, taking the other region's strategy as given. Because decisions are made non-cooperatively, neither region internalizes the environmental damage imposed on the neighboring region through cross-border pollution spillovers. The resulting benchmark is therefore a non-cooperative differential game without any interregional coordination mechanism, and it provides the natural point of comparison for the compensation regimes examined below.

$$J_h^N = \max_{e_h(t),u_h(t)} \int_0^\infty e^{-\rho t}\left[\ln(\alpha_h e_h(t)) + b_h - \frac{c_h}{2}u_h^2(t) - \eta_h x_h(t)\right]dt \quad (7)$$

Proposition 1: Under decentralized decision-making, the Nash equilibrium emission levels and *abatement effort* for each region are, respectively,

$$e_i^{N'} = \frac{1}{\sigma_i}, u_i^{N'} = \frac{l_i\sigma_i}{c_i}, e_j^{N'} = \frac{1}{\sigma_j}, u_j^{N'} = \frac{l_j\sigma_j}{c_j}. \quad (8)$$

where

$$\sigma_i = \frac{\eta_i(\rho + \varphi_j + \phi_j)}{(\rho + \varphi_i + \phi_i)(\rho + \varphi_j + \phi_j) - \varphi_i\varphi_j}, \quad (9)$$

$$\sigma_j = \frac{\eta_j(\rho + \varphi_i + \phi_i)}{(\rho + \varphi_i + \phi_i)(\rho + \varphi_j + \phi_j) - \varphi_i\varphi_j}. \quad (10)$$

The optimal value functions for the two regions under the feedback Nash equilibrium are:

$$V_i^N(x_i, x_j) = K_i^N - \sigma_i x_i - \tau_i x_j, \quad (11)$$

$$V_j^N(x_i, x_j) = K_j^N - \tau_j x_i - \sigma_j x_j. \quad (12)$$

where $\tau_i = \frac{\eta_i\varphi_j}{(\rho+\varphi_i+\phi_i)(\rho+\varphi_j+\phi_j)-\varphi_i\varphi_j}$, $\tau_j = \frac{\eta_j\varphi_i}{(\rho+\varphi_i+\phi_i)(\rho+\varphi_j+\phi_j)-\varphi_i\varphi_j}$.
and

$$\rho K_i^N = \ln\left(\frac{\alpha_i}{\sigma_i}\right) + b_i - 1 + \frac{l_i^2\sigma_i^2}{2c_i} - \frac{\tau_i}{\sigma_j} + \frac{l_j^2\tau_i\sigma_j}{c_j}, \quad (13)$$

$$\rho K_j^N = \ln\left(\frac{\alpha_j}{\sigma_j}\right) + b_j - 1 + \frac{l_j^2\sigma_j^2}{2c_j} - \frac{\tau_j}{\sigma_i} + \frac{l_i^2\tau_j\sigma_i}{c_i}. \quad (14)$$

For the proof, see Supplementary Appendix A.

Proposition 1 characterizes the decentralized feedback Nash equilibrium and therefore provides the benchmark against which all cost-sharing mechanisms are evaluated. The optimal solution shows that each region chooses its emission-generating production and cleanup effort according to its own discounted welfare. In this equilibrium, each region balances its own production benefit, cleanup cost, and pollution damage, but it does not fully account for the environmental damage transmitted to the other region through cross-border pollution diffusion.

Decentralized environmental governance does not necessarily fail because local governments behave irrationally. Rather, it fails because each local government makes rational decisions under incomplete internalization of cross-border damages. The source region has direct control over *emission*-generating production and source-side cleanup, but part of the resulting environmental burden is borne by the affected region. The affected region bears pollution exposure, but it cannot directly determine the source region's source-side decisions. The equilibrium in Proposition 1 therefore reflects an institutional mismatch between control authority and damage exposure.

The optimal strategies in Proposition 1 also indicate that local pollution-control decisions are strategically interdependent even in the absence of formal coordination. Although each region maximizes its own payoff, the state dynamics connect the two regions through pollution accumulation and diffusion. Therefore, the decentralized benchmark should not be interpreted as an efficient governance outcome. It should be interpreted as the baseline outcome generated by fragmented authority. From a managerial perspective, this benchmark justifies the introduction of eco-compensation: a compensation mechanism is needed only if it can improve upon this decentralized incentive structure by changing the effective cost or benefit of cleanup effort.

5 Region-led cost-sharing mechanisms

5.1 Source-led source-cleanup cost-sharing mechanism

Under the source-led cost-sharing mechanism, the pollution source region acts as the Stackelberg leader and proposes a cost-sharing ratio. The affected region is required to share part of the source region's source-side cleanup cost. This mechanism reflects the source region's control advantage in source-side pollution control. Since the source region directly controls emissions and cleanup effort, while the affected region passively bears cross-border pollution damage, the source region can use its control over source-side cleanup to propose a cost-sharing contract.

Let the cost-sharing ratio borne by the affected region be θ. Then the cost-sharing payment from the affected region to the source region is

$$\Gamma_{j\to i}^S(t) = \theta C_i(u_i(t)) = \theta\frac{c_i}{2}u_i^2(t) \quad (15)$$

where $0 \le \theta < 1$. Therefore, the actual source-side cleanup cost borne by the source region is $(1-\theta)\frac{c_i}{2}u_i^2(t)$. When $\theta = 0$, the mechanism reduces to the no-cost-sharing case. When θ increases, the source region's actual cleanup cost decreases, and its incentive to increase cleanup effort becomes stronger.

Given the cost-sharing ratio θ, the objective functional of the source region is

$$J_i^S = \max_{e_i(t),u_i(t)} \int_0^\infty e^{-\rho t}\left[\ln(\alpha_i e_i(t)) + b_i - (1-\theta)\frac{c_i}{2}u_i^2(t) - \eta_i x_i(t)\right]dt \tag{16}$$

The objective function of the affected region is

$$J_j^S = \max_{e_j(t),u_j(t)} \int_0^\infty e^{-\rho t}\left[\ln\left(\alpha_j e_j(t)\right) + b_j - \frac{c_j}{2}u_j^2(t) - \eta_j x_j(t) - \theta\frac{c_i}{2}u_i^2(t)\right]dt \tag{17}$$

The term $\theta c_i u_i^2(t)/2$ is the cost-sharing payment made by the affected region for the source region's source-side cleanup. It should be noted that this payment depends on the source region's cleanup effort $u_i(t)$ rather than on the affected region's own control variable. Therefore, this mechanism mainly changes the source region's source-side cleanup incentive, while it does not directly change the marginal cost of the affected region's own cleanup effort.

The mechanism design problem of the source region as the leader can be written as

$$\max_\theta V_i^S\left(x_i, x_j; \theta\right) \tag{18}$$

$$s.t.\begin{cases} e_h^{S^*}(\theta), u_h^{S^*}(\theta) \in arg\max J_h^S(\theta) \\ V_j^S\left(x_i, x_j; \theta\right) \geq V_j^N\left(x_i, x_j\right) \\ 0 \leq \theta < 1 \end{cases} \tag{19}$$

The first constraint indicates that, given the cost-sharing ratio, the two regions form their optimal responses. The second constraint is the participation constraint of the affected region, which means that the affected region accepts the contract only if its welfare under the cost-sharing mechanism is no lower than its benchmark equilibrium welfare. The third constraint gives the feasible range of the cost-sharing ratio.

Proposition 2: Source-led cost-sharing equilibrium.

Under the source-led cost-sharing mechanism, for any given cost-sharing ratio θ, the feedback equilibrium strategies of the two regions are given by

$$e_i^{S'}(\theta) = \frac{1}{\sigma_i}, u_i^{S'}(\theta) = \frac{l_i\sigma_i}{(1-\theta)c_i}, e_j^{S'}(\theta) = \frac{1}{\sigma_j}, u_j^{S'}(\theta) = \frac{l_j\sigma_j}{c_j}. \tag{20}$$

The equilibrium shows that the cost-sharing mechanism directly affects only the source region's source-side cleanup effort, while it does not change the affected region's own cleanup effort. As θ increases, the actual cleanup cost borne by the source region decreases, and its optimal cleanup effort increases.

The corresponding value functions can be written as

$$V_i^S\left(x_i, x_j; \theta\right) = K_i^S(\theta) - \sigma_i x_i - \tau_i x_j, \tag{21}$$

$$V_j^S\left(x_i, x_j; \theta\right) = K_j^S(\theta) - \tau_j x_i - \sigma_j x_j. \tag{22}$$

The constant terms $K_i^S(\theta)$ and $K_j^S(\theta)$ satisfy:

$$\rho K_i^S(\theta) = \ln\left(\frac{\alpha_i}{\sigma_i}\right) + b_i - 1 + \frac{l_i^2\sigma_i^2}{2(1-\theta)c_i} - \frac{\tau_i}{\sigma_j} + \frac{l_j^2\tau_i\sigma_j}{c_j}, \tag{23}$$

$$\rho K_j^S(\theta) = \ln\left(\frac{\alpha_j}{\sigma_j}\right) + b_j - 1 + \frac{l_j^2\sigma_j^2}{2c_j} - \frac{\tau_j}{\sigma_i} + \frac{l_i^2\tau_j\sigma_i}{(1-\theta)c_i} - \frac{\theta l_i^2\sigma_i^2}{2(1-\theta)^2 c_i}. \tag{24}$$

Furthermore, the optimal cost-sharing ratio chosen by the source region as the Stackelberg leader is

$$\theta^{S^*} = \max\left\{0, 1 - \frac{\eta_i\left(\rho + \varphi_j + \phi_j\right)}{2\eta_j\varphi_i}\right\} \tag{25}$$

For the proof, see Supplementary Appendix B.

Proposition 2 derives the equilibrium under the source-led source-side cleanup cost-sharing mechanism. The optimal solution shows that, for any given cost-sharing ratio, the mechanism directly changes the source region's cleanup decision by reducing its effective source-side cleanup cost. As the affected region shares a larger fraction of the source region's cleanup cost, the source region's optimal cleanup effort increases. However, the affected region's own cleanup effort is not directly changed by this mechanism, because the shared cost is tied to the source region's source-side cleanup rather than to the affected region's local remediation.

This result shows that the source-led mechanism operates through the cleanup-cost channel rather than the emission-control channel. The cost-sharing contract does not directly restrict the source region's emission-generating production. Instead, it lowers the marginal burden of source-side treatment and thereby encourages the source region to increase cleanup effort. The managerial meaning is that source-led eco-compensation should be designed around verifiable source-side pollution-control projects, treatment facilities, or cleanup expenditures. If the shared expenditure cannot be verified, the mechanism may fail to generate the intended behavioral response.

The optimal cost-sharing ratio in Proposition 2 further reveals the role of leadership. Because the source region acts as the Stackelberg leader, it chooses the cost-sharing ratio to maximize its own welfare subject to the affected region's participation constraint. This means that the source region has an incentive to push the ratio toward the highest level that the affected region is willing to accept. In managerial terms, the source region uses its control advantage over source-side cleanup to obtain financial support from the affected region. Such a mechanism can improve source-side treatment, but it may also create distributional tension because the affected region bears part of the cost for pollution generated outside its jurisdiction.

The feasibility of the optimal solution depends on whether the affected region receives enough environmental benefit from stronger source-side cleanup. When pollution transfer from the source region to the affected region is strong, or when the affected region suffers high marginal pollution damage, the affected region obtains greater benefits from source-side treatment and is more likely to accept the contract. Conversely, when cross-border spillovers are weak or the affected region's damage reduction is limited, the participation constraint cannot be satisfied and the optimal arrangement collapses to the no-cost-sharing benchmark. Therefore, the managerial lesson from Proposition 2 is that source-led cost sharing is most appropriate when source-side treatment generates clear and verifiable benefits for the affected region, but it requires safeguards against excessive cost shifting by the source region.

5.2 Affected-region-led source-cleanup subsidy mechanism

Under the affected-region-led cost-sharing mechanism, the affected region acts as the Stackelberg leader and first proposes a cost-sharing ratio. It voluntarily subsidizes the source region's source-side cleanup cost. This mechanism reflects the affected region's active demand for transboundary pollution control. The source region directly controls emissions and source-side cleanup effort, whereas the affected region bears cross-border pollution damage but cannot directly determine the source region's cleanup behavior. Therefore, the affected region can share part of the source region's cleanup cost to induce stronger source-side cleanup efforts and reduce cross-border pollution damage.

Let the subsidy ratio offered by the affected region be θ. Then the cost-sharing payment from the affected region to the source region is

$$\Gamma^V_{j\to i}(t) = \theta C_i(u_i(t)) = \theta\frac{c_i}{2}u_i^2(t) \tag{26}$$

where $0 \le \theta < 1$. Therefore, the actual source-side cleanup cost borne by the source region is $(1-\theta)\frac{c_i}{2}u_i^2(t)$. When $\theta = 0$, the mechanism reduces to the no-cost-sharing case. When θ increases, the actual cleanup cost borne by the source region decreases, and its incentive to increase source-side cleanup effort becomes stronger. Unlike the source-led mechanism, the cost-sharing ratio in the affected-region-led mechanism is not proposed by the source region. Instead, it is determined by the affected region based on the trade-off between reduced pollution damage and subsidy cost.

Given the cost-sharing ratio θ, the objective functional of the source region is

$$J_i^V = \max_{e_i(t),u_i(t)} \int_0^\infty e^{-\rho t}\left[\ln(\alpha_i e_i(t)) + b_i - (1-\theta)\frac{c_i}{2}u_i^2(t) - \eta_i x_i(t)\right]dt \tag{27}$$

The objective function of the affected region is

$$J_j^V = \max_{e_j(t),u_j(t)} \int_0^\infty e^{-\rho t}\left[\ln\left(\alpha_j e_j(t)\right) + b_j - \frac{c_j}{2}u_j^2(t) - \eta_j x_j(t) - \theta\frac{c_i}{2}u_i^2(t)\right]dt \tag{28}$$

The term $\theta c_i u_i^2(t)/2$ denotes the subsidy paid by the affected region for the source region's source-side cleanup. This mechanism changes the source region's source-side cleanup incentive mainly by reducing its actual cleanup cost.

The mechanism design problem of the affected region, as the leader can be written as

$$\max_\theta V_j^V\left(x_i, x_j; \theta\right) \tag{29}$$

$$s.t.\begin{cases} e_h^{V*}(\theta), u_h^{V*}(\theta) \in arg\max J_h^V(\theta) \\ V_h^V\left(x_i, x_j; \theta\right) \ge V_h^N\left(x_i, x_j\right), \mathrm{h} = \mathrm{i,j} \\ 0 \le \theta < 1 \end{cases} \tag{30}$$

Proposition 3: Affected-region-led cost-sharing equilibrium.

Under the affected-region-led cost-sharing mechanism, for any given cost-sharing ratio θ, the feedback equilibrium strategies of the two regions are given by

$$e_i^{V'}(\theta) = \frac{1}{\sigma_i}, u_i^{V'}(\theta) = \frac{l_i\sigma_i}{(1-\theta)c_i}, e_j^{V'}(\theta) = \frac{1}{\sigma_j}, u_j^{V'}(\theta) = \frac{l_j\sigma_j}{c_j}. \tag{31}$$

The corresponding value functions are

$$V_i^V\left(x_i, x_j; \theta\right) = K_i^V(\theta) - \sigma_i x_i - \tau_i x_j, \tag{32}$$

$$V_j^V\left(x_i, x_j; \theta\right) = K_j^V(\theta) - \tau_j x_i - \sigma_j x_j, \tag{33}$$

The constant terms $K_i^V(\theta)$ and $K_j^V(\theta)$ satisfy:

$$\rho K_i^V(\theta) = \ln\left(\frac{\alpha_i}{\sigma_i}\right) + b_i - 1 + \frac{l_i^2\sigma_i^2}{2(1-\theta)c_i} - \frac{\tau_i}{\sigma_j} + \frac{l_j^2\tau_i\sigma_j}{c_j}, \tag{34}$$

$$\rho K_j^V(\theta) = \ln\left(\frac{\alpha_j}{\sigma_j}\right) + b_j - 1 + \frac{l_j^2\sigma_j^2}{2c_j} - \frac{\tau_j}{\sigma_i} + \frac{l_i^2\tau_j\sigma_i}{(1-\theta)c_i} - \frac{\theta l_i^2\sigma_i^2}{2(1-\theta)^2 c_i}. \tag{35}$$

Furthermore, the optimal cost-sharing ratio chosen by the affected region as the Stackelberg leader is

$$\theta^{V*} = \max\left\{0, \frac{2\eta_j\varphi_i - \eta_i\left(\rho + \varphi_j + \phi_j\right)}{2\eta_j\varphi_i + \eta_i\left(\rho + \varphi_j + \phi_j\right)}\right\} \tag{36}$$

For the proof, see Supplementary Appendix C.

Proposition 3 characterizes the equilibrium under an affected-region-led source-side cleanup subsidy mechanism. The optimal solution shows that the affected region chooses a subsidy ratio that balances two opposing effects: the environmental benefit obtained from stronger source-side cleanup and the fiscal cost of providing the subsidy. Because the subsidy lowers the source region's effective cleanup cost, the source region responds by increasing its cleanup effort. As a result, cross-border pollution is reduced and the affected region experiences lower environmental damage.

The optimal subsidy ratio has a clear managerial interpretation. It represents the affected region's willingness to pay for pollution reduction that it cannot directly achieve through its own regulatory authority. In many transboundary pollution problems, the affected region suffers environmental losses but lacks control over the source region's production and treatment decisions. The optimal solution indicates that, under such circumstances, financial incentives can serve as an indirect governance tool. Rather than attempting to influence pollution outcomes through costly local remediation alone, the affected region can improve environmental quality by encouraging additional cleanup at the pollution source.

The equilibrium also highlights the importance of benefit-cost matching in eco-compensation design. A higher subsidy is justified only when additional source-side cleanup generates sufficiently large reductions in cross-border pollution damage. Therefore, the effectiveness of the mechanism depends on the environmental return generated by each unit of subsidized cleanup effort. From a managerial perspective, this implies that subsidy programs should prioritize source-side treatment activities with clear, measurable, and significant spillover benefits. If the environmental gains from source-side cleanup are limited, increasing the subsidy ratio will generate little welfare improvement and may lead to inefficient use of public resources.

Another implication of the optimal solution is that beneficiary-financed eco-compensation is inherently voluntary and therefore constrained by the affected region's expected gains. The affected region will choose a positive subsidy only when the reduction in environmental damage outweighs the associated fiscal burden. Consequently, successful implementation requires credible information on pollution transmission, treatment effectiveness, and environmental benefits. Without reliable evidence that source-side cleanup produces meaningful improvements in environmental quality, the affected region has little incentive to provide financial support.

6 Government-led cost-sharing mechanisms

6.1 Government-led polluter-pays cost-compensation mechanism

Under the government-led polluter-pays cost-compensation mechanism, the upper-level government acts as the Stackelberg leader and first determines the cost-compensation ratio. The source region is required to compensate the affected region for part of its local pollution-control cost. This mechanism reflects the polluter-pays principle. Since the source region's production and emissions generate cross-border pollution damage to the affected region, the source region should bear part of the affected region's local cleanup cost.

Unlike the two region-led mechanisms, the cost-compensation ratio under the government-led mechanism is not determined by the source region or the affected region based on their own welfare maximization. Instead, it is set by the upper-level government from the perspective of maximizing aggregate social welfare across the two regions. Since the upper-level government has institutional authority and coordination power, local participation constraints are not imposed in this mechanism. In other words, whether the source region or the affected region obtains a welfare improvement relative to the benchmark equilibrium is not a necessary condition for implementing the mechanism.

Let the compensation ratio determined by the upper-level government be θ. Then the cost compensation paid by the source region to the affected region is

$$\Gamma^{G1}_{i\to j}(t) = \theta C_j\left(u_j(t)\right) = \theta \frac{c_j}{2} u_j^2(t) \tag{37}$$

where $0 \le \theta < 1$. Therefore, the actual local cleanup cost borne by the affected region is $(1-\theta)\frac{c_j}{2}u_j^2(t)$, while the source region bears its own source-side cleanup cost and additionally pays part of the affected region's local cleanup cost. When $\theta = 0$, the mechanism reduces to the no-compensation case. When θ increases, the actual local cleanup cost borne by the affected region decreases, and its incentive to increase local cleanup effort becomes stronger.

Given the compensation ratio θ, the objective functional of the source region is

$$J_i^{G1} = \max_{e_i(t),u_i(t)} \int_0^\infty e^{-\rho t}\left[\ln\left(\alpha_i e_i(t)\right) + b_i - \frac{c_i}{2}u_i^2(t) - \eta_i x_i(t) - \theta\frac{c_j}{2}u_j^2(t)\right]dt. \tag{38}$$

The objective functional of the affected region is

$$J_j^{G1} = \max_{e_j(t),u_j(t)} \int_0^\infty e^{-\rho t}\left[\ln\left(\alpha_j e_j(t)\right) + b_j - (1-\theta)\frac{c_j}{2}u_j^2(t) - \eta_j x_j(t)\right]dt. \tag{39}$$

Under this mechanism, the upper-level government first chooses the cost-compensation ratio. The two regions then choose their emission levels and cleanup efforts given θ. The upper-level government anticipates the optimal responses of both regions and maximizes their aggregate social welfare. Therefore, the government's mechanism design problem can be written as

$$\max_{\theta}\left[V_i^{G1}\left(x_i, x_j; \theta\right) + V_j^{G1}\left(x_i, x_j; \theta\right)\right] \tag{40}$$

$$s.t.\begin{cases} \left(e_h^{G1*}(\theta), u_h^{G1*}(\theta)\right) \in \arg\max J_h^{G1}(\theta), h = i, j, \\ 0 \le \theta < 1. \end{cases} \tag{41}$$

The first constraint is the optimal-response constraint of the two regions given the cost-compensation ratio. It means that the source region and the affected region choose their emission levels and cleanup efforts according to their own objective functions. The second constraint gives the feasible range of the compensation ratio.

Proposition 4: Government-led polluter-pays cost-compensation equilibrium.

Under the government-led polluter-pays cost-compensation mechanism, for any given compensation ratio θ, the feedback equilibrium strategies of the two regions are given by

$$e_i^{G1'}(\theta) = \frac{1}{\sigma_i}, u_i^{G1'}(\theta) = \frac{l_i\sigma_i}{c_i}, e_j^{G1'}(\theta) = \frac{1}{\sigma_j}, u_j^{G1'}(\theta) = \frac{l_j\sigma_j}{(1-\theta)c_j}. \tag{42}$$

The corresponding value functions are

$$V_i^{G1}\left(x_i, x_j; \theta\right) = K_i^{G1}(\theta) - \sigma_i x_i - \tau_i x_j, \tag{43}$$

$$V_j^{G1}\left(x_i, x_j; \theta\right) = K_j^{G1}(\theta) - \tau_j x_i - \sigma_j x_j. \tag{44}$$

and

$$\rho K_i^{G1}(\theta) = \ln\left(\frac{\alpha_i}{\sigma_i}\right) + b_i - 1 + \frac{l_i^2\sigma_i^2}{2c_i} - \frac{\tau_i}{\sigma_j} + \frac{l_j^2\tau_i\sigma_j}{(1-\theta)c_j} - \frac{\theta l_j^2\sigma_j^2}{2(1-\theta)^2 c_j} \tag{45}$$

$$\rho K_j^{G1}(\theta) = \ln\left(\frac{\alpha_j}{\sigma_j}\right) + b_j - 1 + \frac{l_j^2\sigma_j^2}{2(1-\theta)c_j} - \frac{\tau_j}{\sigma_i} + \frac{l_i^2\tau_j\sigma_i}{c_i}. \tag{46}$$

Furthermore, the optimal compensation ratio chosen by the upper-level government to maximize aggregate social welfare is

$$\theta^{G1*} = \frac{\eta_i\varphi_j}{\eta_i\varphi_j + \eta_j\left(\rho + \varphi_i + \phi_i\right)} \tag{47}$$

For the proof, see Supplementary Appendix D.

Proposition 4 derives the equilibrium under the government-led polluter-pays cost-compensation mechanism. The optimal solution shows that the upper-level government requires the source region to compensate part of the affected region's local cleanup cost. This reduces the affected region's effective marginal cost of local remediation and therefore increases the affected region's cleanup effort. At the same time, the mechanism does not directly increase the source region's source-side cleanup effort, because the shared cost is the affected region's local cleanup cost rather than the source region's own treatment cost.

This result clarifies the operational meaning of the polluter-pays principle in this model. Polluter-pays compensation is not simply a transfer from the source region to the affected region. Its behavioral effect depends on whether the compensation changes the affected region's marginal incentive to clean up. If compensation is linked to verifiable local remediation expenditure, it can induce the affected region to undertake more local treatment. If compensation is paid as an unconditional fiscal transfer, however, the incentive effect may be weak. Therefore, the managerial value of this mechanism depends on tying compensation to observable remediation activities.

The optimal compensation ratio chosen by the upper-level government is based on aggregate welfare rather than on the private objective of either region. This distinguishes government-led compensation from region-led bargaining. The government can impose a ratio even when one region would not voluntarily agree to it, because the purpose is to improve the overall welfare of the transboundary system. The optimal solution implies that the compensation ratio should be higher when affected-region cleanup generates stronger welfare benefits beyond the affected region itself. In particular, if pollution in the affected region creates indirect environmental or welfare losses for the source region, then supporting affected-region remediation becomes more valuable from the perspective of aggregate welfare.

6.2 Government-led beneficiary-pays source-cleanup cost-sharing mechanism

Under the government-led beneficiary-pays source-cleanup cost-sharing mechanism, the upper-level government acts as the Stackelberg leader and first determines the cost-sharing ratio. The affected region is required to share part of the source region's source-side cleanup cost. This mechanism reflects the beneficiary-pays principle. Although the affected region is not the pollution generator, it benefits from stronger source-side cleanup and lower cross-border pollution damage. Therefore, the upper-level government can require the affected region to share part of the source region's cleanup cost, thereby reducing the source region's actual cleanup cost and inducing stronger source-side cleanup effort.

Unlike the region-led mechanisms, the cost-sharing ratio under the government-led mechanism is not proposed by a local region based on its own welfare-maximization objective. Instead, it is determined by the upper-level government from the perspective of maximizing aggregate social welfare across the two regions. Since the upper-level government has institutional authority and coordination power, local participation constraints are not imposed in this mechanism. That is, whether the source region or the affected region obtains a welfare improvement relative to the benchmark equilibrium is not a necessary condition for implementing the mechanism.

Let the cost-sharing ratio determined by the upper-level government be θ. Then the cost-sharing payment from the affected region to the source region is

$$\Gamma^{G2}_{j\to i}(t) = \theta C_i(u_i(t)) = \theta \frac{c_i}{2} u_i^2(t), \tag{48}$$

where $0 \le \theta < 1$. Therefore, the actual source-side cleanup cost borne by the source region is $(1-\theta)\frac{c_i}{2}u_i^2(t)$, while the affected region bears its own local cleanup cost and additionally pays part of the source region's source-side cleanup cost. When $\theta = 0$, the mechanism reduces to the no-cost-sharing case. When θ increases, the actual source-side cleanup cost borne by the source region decreases, and its incentive to increase source-side cleanup effort becomes stronger.

Given the cost-sharing ratio θ, the objective functional of the source region is

$$J_i^{G2} = \max_{e_i(t),u_i(t)} \int_0^\infty e^{-\rho t}\left[\ln(\alpha_i e_i(t)) + b_i - (1-\theta)\frac{c_i}{2}u_i^2(t) - \eta_i x_i(t)\right]dt. \tag{49}$$

The objective functional of the affected region is

$$J_j^{G2} = \max_{e_j(t),u_j(t)} \int_0^\infty e^{-\rho t}\left[\ln\left(\alpha_j e_j(t)\right) + b_j - \frac{c_j}{2}u_j^2(t) - \eta_j x_j(t) - \theta\frac{c_i}{2}u_i^2(t)\right]dt. \tag{50}$$

The term $\theta c_i u_i^2(t)/2$ denotes the cost-sharing payment made by the affected region for the source region's source-side cleanup. This cost-sharing item depends on the source region's cleanup effort rather than on the affected region's own control variable. Therefore, this mechanism mainly strengthens the source region's cleanup incentive by reducing its actual source-side cleanup cost, while it does not directly change the marginal cost of the affected region's own cleanup effort.

Under this mechanism, the upper-level government first chooses the cost-sharing ratio. The two regions then choose their emission levels and cleanup efforts given θ. The upper-level government anticipates the optimal responses of both regions and maximizes their aggregate social welfare. Therefore, the government's mechanism design problem can be written as

$$\max_\theta \left[V_i^{G2}\left(x_i, x_j; \theta\right) + V_j^{G2}\left(x_i, x_j; \theta\right)\right] \tag{51}$$

$$s.t. \begin{cases} \left(e_h^{G2*}(\theta), u_h^{G2*}(\theta)\right) \in \arg max\, J_h^{G2}(\theta), h = i, j, \\ 0 \le \theta < 1. \end{cases} \tag{52}$$

Proposition 5: Government-led beneficiary-pays source-cleanup cost-sharing equilibrium.

Under the government-led beneficiary-pays source-cleanup cost-sharing mechanism, for any given cost-sharing ratio θ, the feedback equilibrium strategies of the two regions are given by

$$e_i^{G2'}(\theta) = \frac{1}{\sigma_i}, u_i^{G2'}(\theta) = \frac{l_i\sigma_i}{(1-\theta)c_i}, e_j^{G2'}(\theta) = \frac{1}{\sigma_j}, u_j^{G2'}(\theta) = \frac{l_j\sigma_j}{c_j}. \tag{53}$$

The corresponding value functions are

$$V_i^{G2}\left(x_i, x_j; \theta\right) = K_i^{G2}(\theta) - \sigma_i x_i - \tau_i x_j, \tag{54}$$

$$V_j^{G2}\left(x_i, x_j; \theta\right) = K_j^{G2}(\theta) - \tau_j x_i - \sigma_j x_j. \tag{55}$$

and

$$\rho K_i^{G2}(\theta) = \ln\left(\frac{\alpha_i}{\sigma_i}\right) + b_i - 1 + \frac{l_i^2\sigma_i^2}{2(1-\theta)c_i} - \frac{\tau_i}{\sigma_j} + \frac{l_j^2\tau_i\sigma_j}{c_j}, \tag{56}$$

$$\rho K_j^{G2}(\theta) = \ln\left(\frac{\alpha_j}{\sigma_j}\right) + b_j - 1 + \frac{l_j^2\sigma_j^2}{2c_j} - \frac{\tau_j}{\sigma_i} + \frac{l_i^2\tau_j\sigma_i}{(1-\theta)c_i} - \frac{\theta l_i^2\sigma_i^2}{2(1-\theta)^2 c_i} \tag{57}$$

Furthermore, the optimal cost-sharing ratio chosen by the upper-level government to maximize aggregate social welfare is

$$\theta^{G2^*} = \frac{\eta_j\varphi_i}{\eta_i(\rho + \varphi_j + \phi_j) + \eta_j\varphi_i} \tag{58}$$

For the proof, see Supplementary Appendix E.

Proposition 5 characterizes the equilibrium outcome under a government-led beneficiary-pays source-side cleanup cost-sharing mechanism. The equilibrium solution indicates that the upper-level government assigns part of the source region's source-side cleanup cost to the affected region. By lowering the source region's effective marginal cost of treatment, this arrangement tends to encourage greater source-side cleanup effort. In contrast, the affected region's own cleanup decision is not directly influenced by the cost-sharing rule, since the shared expenditure is linked to the source region's treatment activity rather than to local remediation undertaken by the affected region.

The result provides insight into how the beneficiary-pays principle may be operationalized within an eco-compensation framework. Although the affected region is not responsible for generating the pollution, it may receive environmental benefits from enhanced source-side treatment through reduced transboundary pollution inflows. From this perspective, government coordination can facilitate a cost-sharing arrangement in which beneficiaries contribute to pollution-control activities that generate measurable environmental gains for them. Rather than serving as a punitive instrument, the mechanism functions as an incentive-based arrangement that reallocates treatment costs in a manner intended to better align costs and benefits across regions.

The equilibrium cost-sharing ratio reflects aggregate welfare considerations rather than the objectives of either region alone. The analytical results suggest that the optimal ratio tends to be higher when the affected region experiences greater marginal environmental damage or when pollution transmission from the source region is more pronounced. Under such circumstances, source-side cleanup is likely to generate larger environmental benefits for the affected region, thereby strengthening the rationale for beneficiary participation in treatment costs. By contrast, the optimal ratio may be lower when the source region already faces substantial incentives for cleanup due to its own environmental damages, or when factors such as natural pollutant dissipation and discounting reduce the marginal value of additional treatment. Accordingly, the equilibrium highlights the importance of linking cost-sharing arrangements to the magnitude of the environmental benefits generated by source-side cleanup.

A government-led beneficiary-pays mechanism may be particularly relevant when source-side treatment is relatively more cost-effective than remediation undertaken in the affected region and when the direction of pollution spillovers can be identified with reasonable confidence. Relative to source-led cost-sharing arrangements, government involvement may help mitigate concerns that bargaining power could lead to disproportionate cost burdens on the affected region. Relative to affected-region-led subsidy schemes, government coordination may support cost-sharing ratios that more closely reflect system-wide welfare considerations when voluntary contributions fall below the level implied by aggregate benefits. Taken together, Proposition 5 suggests that upper-level governmental coordination can, under certain conditions, facilitate the design and implementation of source-side cost-sharing arrangements when decentralized bargaining alone may not fully internalize transboundary environmental externalities.

7 Numerical analysis

This section conducts numerical simulations to illustrate the theoretical results and examine the sensitivity of the optimal cost-sharing ratios. The purpose is not to calibrate the model to a specific basin or airshed, but to construct an empirically interpretable asymmetric transboundary pollution setting. This setting captures a common governance structure: the source region obtains larger production benefits from pollution-generating activities, while the affected region suffers greater marginal environmental damage and faces higher local cleanup costs.

The time unit is 1 year. The pollution stocks $x_i(t)$ and $x_j(t)$ are interpreted as SO_2-equivalent annual mean pollution concentrations, measured in $\mu g/m^3$. The baseline parameters are set as follows Table 2.

The parameter setting reflects two considerations. First, $\varphi_i > \varphi_j$ and $\eta_j > \eta_i$, which describe an asymmetric transboundary pollution structure: pollution mainly flows from the source region to the affected region, and the affected region is more sensitive to pollution exposure. Second, $c_j > c_i$, which reflects that local remediation in the affected region is more costly than source-side treatment. This is consistent with the policy intuition that source treatment is often more efficient than downstream remediation.

The numerical design follows the scenario-comparison approach commonly used in ecological compensation and differential game studies (Ding et al., 2022; Sheng and Yang, 2024; Xu et al., 2024). Recent studies compare benchmark cases with alternative compensation or cooperative governance scenarios, such as no cost sharing, one-way or two-way compensation, and collaborative governance. In the same spirit, this paper uses numerical simulations to compare mechanisms and test parameter sensitivity, rather than to provide exact empirical calibration.

7.1 Relative externality ratios

To organize the sensitivity analysis, two relative externality ratios are defined:

$$R_{ij} = \frac{\tau_j}{\sigma_i}, R_{ji} = \frac{\tau_i}{\sigma_j}. \tag{59}$$

R_{ij} measures the strength of the source region's external effect on the affected region relative to the source region's own marginal pollution damage. A larger R_{ij} means that source-side treatment creates greater environmental benefits for the affected region.

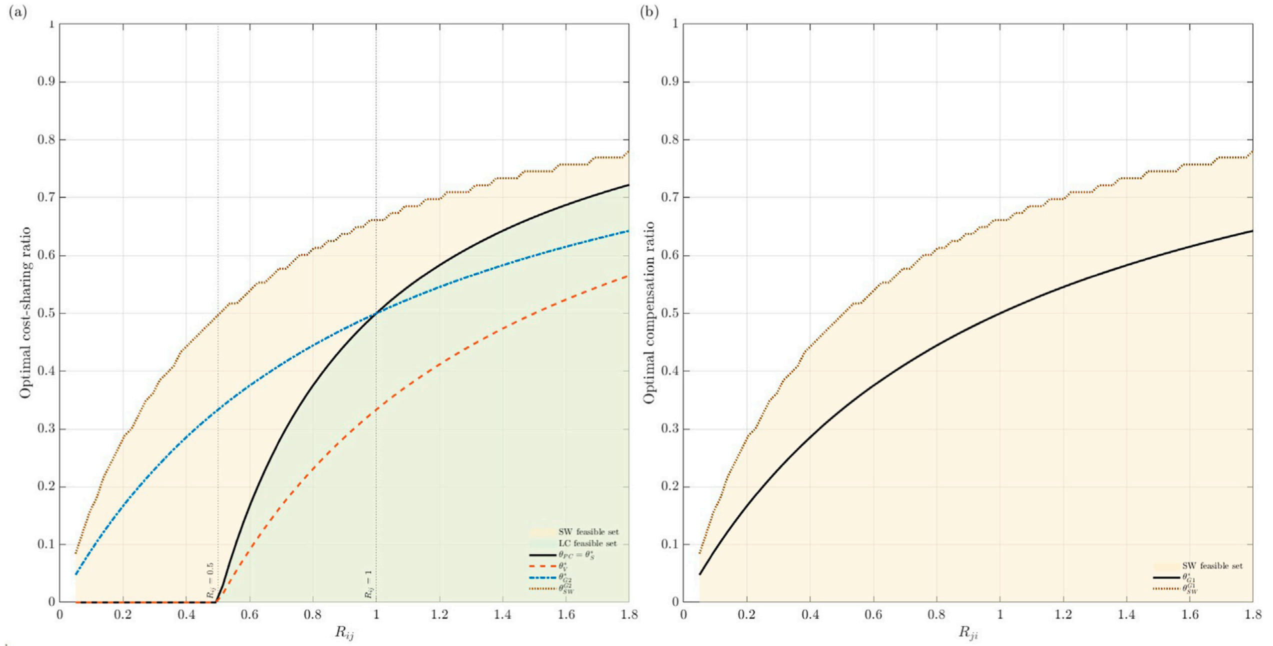

FIGURE 1
Sensitivity of optimal cost-sharing ratios. Panel **(a)** reports source-treatment cost-sharing mechanisms. Panel **(b)** reports the government-led polluter-pays compensation mechanism.

Similarly, R_{ji} measures the reverse externality from the affected region to the source region.

These two ratios are useful because the optimal cost-sharing ratios can be expressed compactly as

$$\theta_S^* = \max\left\{0, 1 - \frac{1}{2R_{ij}}\right\}, \theta_V^* = \max\left\{0, \frac{2R_{ij} - 1}{2R_{ij} + 1}\right\}, \theta_{G1}^* = \frac{R_{ji}}{1 + R_{ji}}, \theta_{G2}^* = \frac{R_{ij}}{1 + R_{ij}}. \quad (60)$$

Thus, R_{ij} determines the strength of source-treatment cost-sharing mechanisms, including source-led cost sharing, affected-region-led subsidy, and government-led beneficiary-pays cost sharing. By contrast, R_{ji} determines the strength of the government-led polluter-pays compensation mechanism. In the sensitivity analysis, R_{ij} and R_{ji} are varied over $[0.05, 1.80]$, while the cost-sharing ratio θ is varied over $[0, 0.95]$.

7.2 Sensitivity of optimal cost-sharing ratios

Figure 1 reports how the optimal cost-sharing ratios respond to changes in the relative externality ratios. In Panel (a), the horizontal axis is R_{ij}, and the vertical axis is the optimal source-treatment cost-sharing ratio. The green shaded area denotes the locally coordinated feasible set, where the affected region's participation constraint is satisfied. The amber shaded area denotes the social-welfare feasible set under government-led coordination. The black solid curve represents the participation boundary $\theta_{PC} = \theta_S^*$. The orange dashed curve represents the affected-region-led ratio θ_V^*, and the blue dash-dotted curve represents the government-led beneficiary-pays ratio θ_{G2}^*. The dotted brown curve represents the social-welfare feasible upper boundary.

Panel (a) shows that $R_{ij} = 0.5$ is the key threshold for region-led source-treatment cost sharing. When $R_{ij} \leq 0.5$, the source-to-affected-region externality is too weak to support a positive locally coordinated cost-sharing contract; both θ_S^* and θ_V^* are zero. When $R_{ij} > 0.5$, positive local cost sharing becomes feasible, and both θ_S^* and θ_V^* increase with R_{ij}. This means that a stronger source-to-affected-region externality increases the affected region's willingness to finance source-side treatment.

The government-led beneficiary-pays ratio $\theta_{G2}^* = R_{ij}/(1 + R_{ij})$ is positive for any $R_{ij} > 0$, because the upper-level government chooses the ratio from the perspective of aggregate welfare and does not require voluntary participation by local regions. The ranking of the ratios depends on the magnitude of R_{ij}. When $0.5 < R_{ij} < 1$, $\theta_{G2}^* > \theta_S^* > \theta_V^*$. When $R_{ij} > 1$, $\theta_S^* > \theta_{G2}^* > \theta_V^*$. This ranking shows that leadership structure affects the cost-sharing outcome. Source leadership allows the source region to push the ratio to the affected region's participation boundary. Victim leadership generates a lower ratio because the affected region internalizes its own subsidy cost. Government leadership produces a social-welfare-oriented ratio that differs from both region-led ratios.

Panel (b) reports the government-led polluter-pays compensation mechanism. The black solid curve represents θ_{G1}^*, and the dotted brown curve represents the social-welfare feasible upper boundary. The optimal compensation ratio $\theta_{G1}^* = \frac{R_{ji}}{1+R_{ji}}$ is increasing and concave in R_{ji}. Thus, the upper-level government requires a higher compensation ratio when the affected region's local cleanup creates larger indirect benefits for the source region. However, when R_{ji} is small, the polluter-pays compensation ratio remains limited. This is because the mechanism mainly strengthens

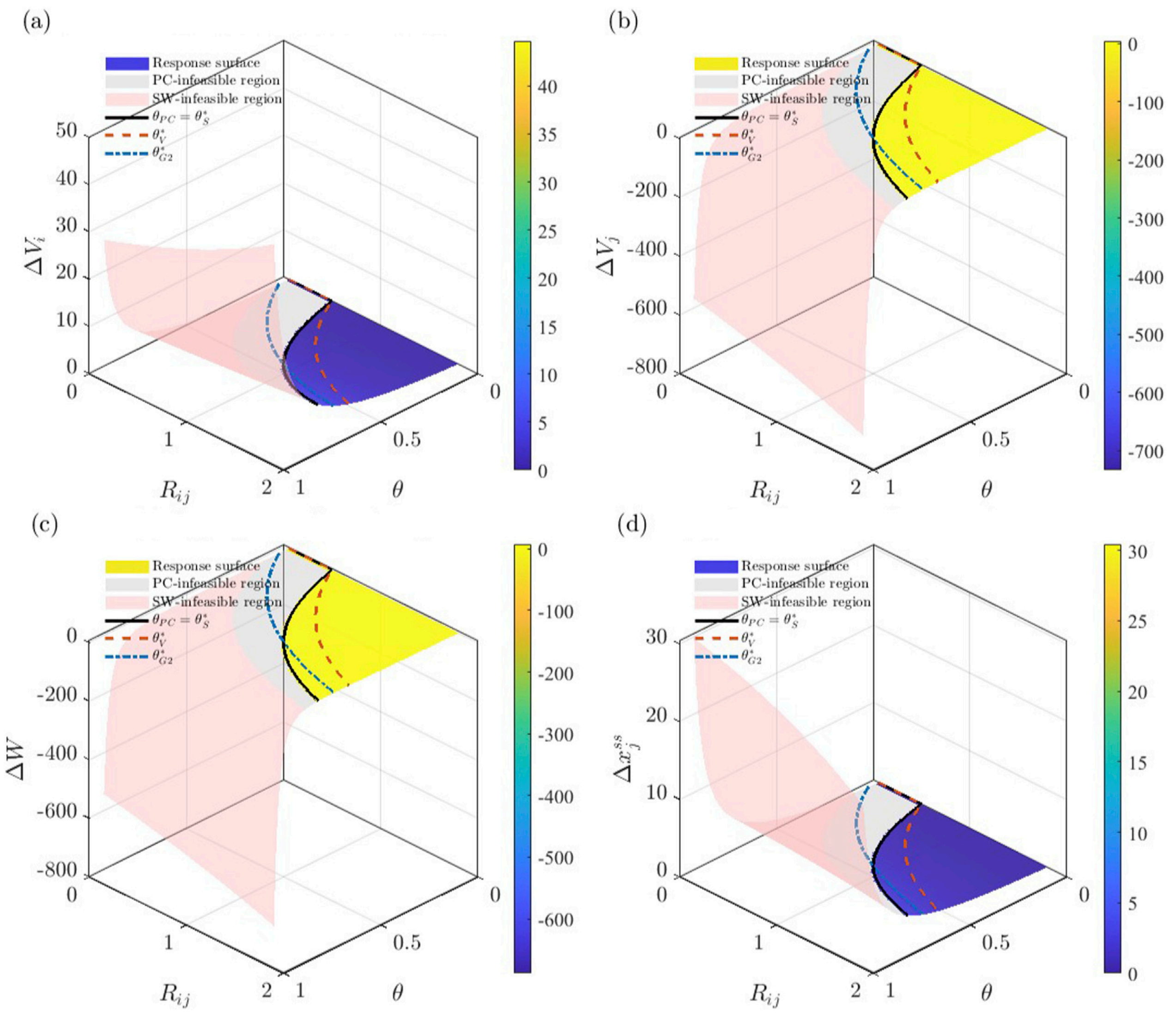

FIGURE 2
Welfare and pollution responses under source-treatment cost-sharing mechanisms. **(a)** Source-region value change. **(b)** Affected-region value change. **(c)** Aggregate welfare change. **(d)** Steady-state pollution reduction.

the affected region's local cleanup incentive rather than directly stimulating source-side treatment.

7.3 Welfare and pollution responses

Figure 2 reports the response surfaces for source-treatment cost-sharing mechanisms. The horizontal axes are the cost-sharing ratio θ and the relative externality ratio R_{ij}. The colored surface represents the response value. The gray area denotes the participation-constraint-infeasible region, where the affected region would not voluntarily accept the cost-sharing arrangement. The red shaded area denotes the social-welfare-infeasible region, where aggregate welfare gain is negative. The black solid curve represents the source-led boundary $\theta_{PC} = \theta_S^*$, the orange dashed curve represents θ_V^*, and the blue dash-dotted curve represents θ_{G2}^*.

Figure 2 shows that source-treatment mechanisms become more effective when R_{ij} and θ are high. A larger R_{ij} means that source-side treatment produces greater environmental benefits for the affected region. A higher θ reduces the actual treatment cost borne by the source region and therefore increases its source-side treatment effort. As a result, the steady-state pollution stock in the affected region declines, as shown in Panel (d).

The welfare effects are asymmetric across regions. Panel (a) shows that the source region tends to gain from source-treatment cost sharing because part of its treatment cost is shifted to the affected region. Panel (b) shows that the affected-region gain is constrained by the participation boundary. Once θ exceeds the locally feasible region, the affected region's payment burden dominates its pollution-reduction benefit. Panel (c) shows that aggregate welfare gain is positive only within part of the parameter space, which explains why the government-led social-welfare feasible set is smaller than the full parameter domain.

The comparison of the three optimal-ratio curves confirms the role of leadership. The source-led curve lies on the affected

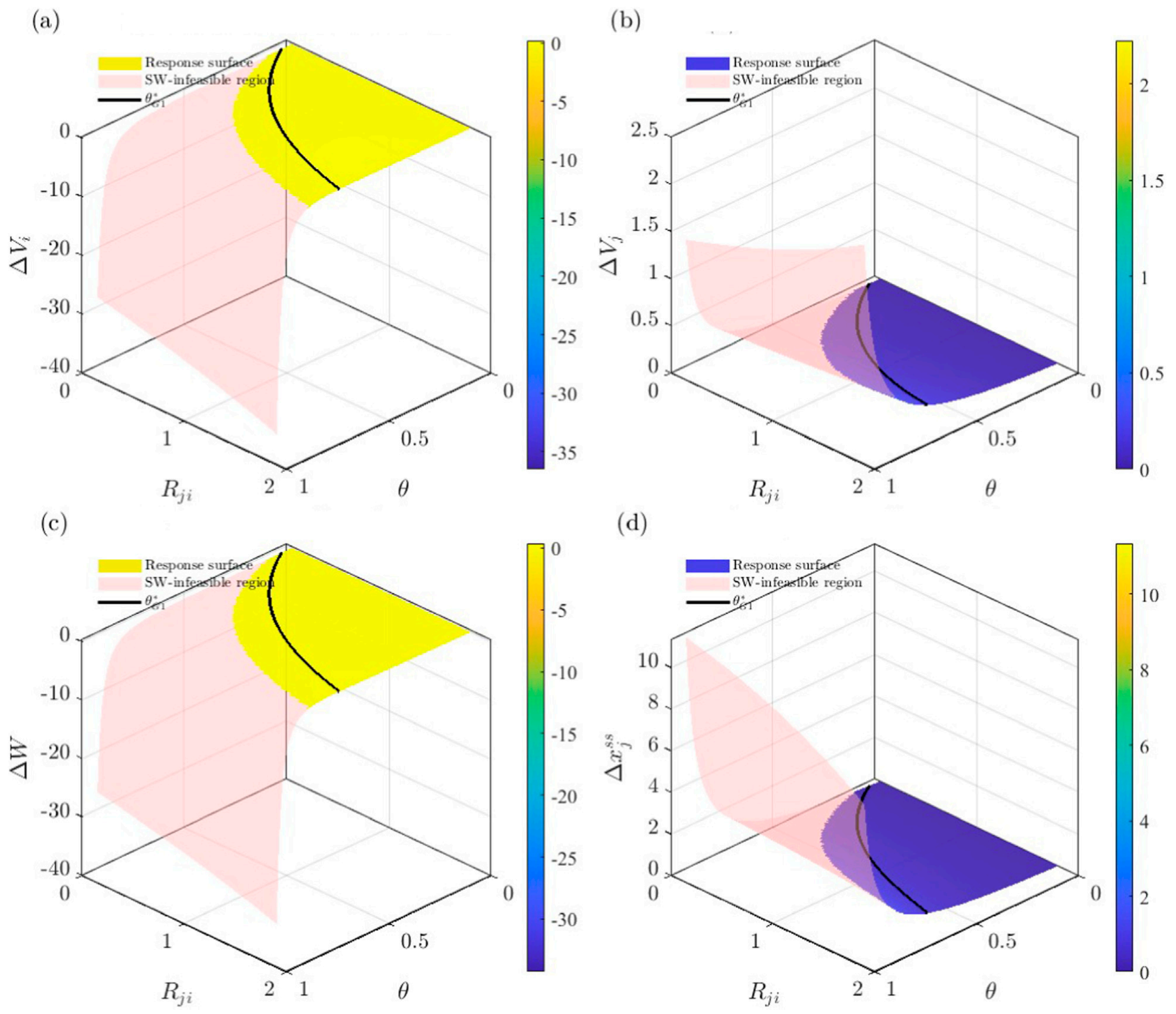

FIGURE 3
Welfare and pollution responses under government-led polluter-pays compensation. Panels **(a–d)** report source-region value gain, affected-region value gain, aggregate welfare gain, and steady-state pollution reduction, respectively.

region's participation boundary, meaning that the source region extracts the maximum acceptable cost-sharing ratio. The affected-region-led curve is lower because the affected region chooses a more conservative ratio after internalizing the subsidy cost. The government-led beneficiary-pays curve reflects aggregate welfare rather than either region's private payoff. Therefore, the same source-treatment cost-sharing direction can generate different equilibrium outcomes depending on who determines the ratio.

Figure 3 reports the response surfaces for the government-led polluter-pays compensation mechanism. The horizontal axes are θ and R_{ji}. The colored surface represents the response value, the red shaded area denotes the social-welfare-infeasible region, and the black solid curve represents the optimal compensation ratio θ^*_{G1}.

This mechanism works through a different channel from source-treatment cost sharing. It reduces the affected region's local cleanup cost and increases its local cleanup effort. As R_{ji} rises, the social value of affected-region cleanup becomes larger, and the optimal compensation ratio increases. This pattern is reflected in the upward movement of θ^*_{G1}.

However, Figure 3 also shows the limitation of polluter-pays compensation. Panel (a) indicates that the source region may experience a value loss because it bears part of the affected region's local cleanup cost. Panel (b) shows that the affected region benefits from the reduced cleanup burden. Panel (c) indicates that aggregate welfare improvement is conditional: excessive compensation may enter the social-welfare-infeasible region. Panel (d) shows that steady-state pollution reduction increases with θ and R_{ji}, but the mechanism does not directly reduce the source region's source-side treatment cost.

Comparing Figures 2, 3 highlights the difference between the two government-led mechanisms. Beneficiary-pays source-treatment cost sharing is more relevant when the dominant externality runs from the source region to the affected region, as captured by a high R_{ij}. Polluter-pays compensation is more relevant when affected-region local cleanup generates substantial spillover benefits to the source region, as captured by a high R_{ji}. In the asymmetric benchmark considered here, the source-to-affected-region externality is the main channel, so source-treatment cost sharing has stronger implications for transboundary pollution reduction.

Overall, the numerical results support the theoretical propositions. Cost-sharing eco-compensation is not universally effective. Its performance depends on the strength and direction of the externality, the leadership structure, and the relevant feasibility constraint. When source-to-affected-region externality is strong and verified cleanup costs can be shared, cost sharing can serve as a useful complementary instrument for decentralized transboundary pollution control. However, when the main policy objective is to directly discipline emission decisions, emission taxes or tradable permits remain more appropriate instruments.

8 Conclusions and managerial implications

8.1 Conclusions

The core problem in transboundary pollution control lies in the separation between control authority and damage exposure. The source region controls emissions and pollution-control efforts, but does not fully bear the cross-border environmental damage. The affected region suffers from this damage, but has limited ability to determine the behavior of the source region. This mismatch weakens the incentives for pollution control under decentralized governance and provides the institutional basis for interregional eco-compensation and cost-sharing arrangements.

To analyze this problem, this study develops a continuous-time differential game model between a source region and an affected region. The model incorporates emissions, pollution-control effort, pollution stock accumulation, and cross-border diffusion. A decentralized benchmark without coordination is first established, and four cost-sharing eco-compensation mechanisms are then compared: source-region-led source-treatment cost sharing, affected-region-led source-treatment subsidy, government-led polluter-pays compensation, and government-led beneficiary-pays cost sharing. The framework distinguishes three key elements: the object of cost sharing, the leading actor, and the incentive channel.

The main conclusions are as follows.

First, cost sharing improves outcomes mainly by reducing cleanup costs rather than by directly constraining emissions. Cost sharing does not directly change emission decisions. Instead, it lowers the marginal cost of pollution-control activities and thereby increases cleanup effort. Source-treatment cost sharing reduces the source region's cleanup cost and increases its treatment effort, while polluter-pays compensation reduces the affected region's remediation cost and increases its local cleanup effort. Therefore, the effectiveness of the mechanism depends on whether the subsidized activity is the one that generates cross-border environmental benefits. This instrument is not a substitute for emission taxes or emissions trading, but a complementary tool when cross-regional pricing and unified enforcement are difficult to implement.

Second, leadership structure affects both the cost-sharing ratio and the distributional outcome. When the source region acts as the leader, it uses its control advantage to push the cost-sharing ratio toward the affected region's participation boundary. This mechanism can strengthen source-side treatment, but it may also shift more costs to the affected region. When the affected region acts as the leader, it subsidizes the source region in order to reduce environmental damage, but because it internalizes its own payment burden, the resulting sharing ratio is more conservative. When an upper-level government acts as the leader, the ratio is determined from the perspective of aggregate welfare. Government leadership can therefore play a coordinating role when participation constraints, asymmetric bargaining power, or distributional conflicts hinder local coordination.

Third, the effectiveness of the mechanism is conditional. The numerical results show that the optimal cost-sharing ratio increases with the relevant relative externality ratio. Source-treatment cost sharing is more suitable when the dominant externality runs from the source region to the affected region. Polluter-pays compensation is more suitable when remediation in the affected region generates substantial reverse benefits for the source region. If the direction of compensation is inconsistent with the direction of the externality, or if the environmental benefit is insufficient to offset the payment burden, the mechanism may degenerate into fiscal redistribution and fail to improve environmental outcomes.

8.2 Policy implications: an operational framework for mechanism selection

The above conclusions indicate that eco-compensation design should be translated into an operational selection procedure. Policymakers should answer the following questions in sequence: What is the direction of the externality? Is the externality strong enough to support voluntary compensation? Does the object of compensation correspond to an effective pollution-control activity? Are both the participation constraint and aggregate welfare feasibility satisfied? Is upper-level government intervention needed when local bargaining fails? Table 3 summarizes the operational rules for mechanism selection.

The policy logic reflected in Table 3 can be further summarized as follows.

First, policymakers should identify the direction of the externality. The relative externality ratios and R_{ji} can be used as diagnostic indicators. A high R_{ij} suggests that source-region treatment generates substantial benefits for the affected region, in which case source-treatment cost sharing or a beneficiary-pays mechanism should be prioritized. A high R_{ji} suggests that remediation in the affected region generates reverse benefits for the source region, in which case polluter-pays compensation is more appropriate. The direction of compensation must therefore be consistent with the direction of the externality; otherwise, the mechanism is unlikely to create effective pollution-control incentives.

Second, threshold conditions should be made explicit rather than replaced by a general call for expanding eco-compensation. For locally negotiated source-treatment cost sharing, $R_{ij} = 0.5$ is

TABLE 3 Operational rules for selecting eco-compensation mechanisms.

Policy diagnostic question	Model indicator	Operational judgment	Recommended mechanism	Institutional arrangement
Does source-region treatment significantly benefit the affected region?	R_{ij}	$R_{ij} > 0.5$	Source-treatment cost sharing/ beneficiary-pays mechanism	The downstream or beneficiary region compensates upstream treatment projects; payments should be linked to cross-border water quality, treatment project lists, and third-party verification
Is the source-side externality weak?	$R_{ij} \leq 0.5$	Voluntary local cost sharing is infeasible	Horizontal compensation should not be imposed	Priority should be given to emission standards, enforcement, targeted treatment programs, or upper-level fiscal support
Does local remediation in the affected region generate reverse benefits for the source region?	High R_{ji}	Local remediation has cross-border benefits	Polluter-pays compensation	The source region compensates remediation expenditure in the affected region
Can local governments reach an agreement and is aggregate welfare positive?	Participation constraint + $\Delta W \geq 0$	A horizontal agreement is feasible	Local bargaining	The agreement should specify the sharing ratio, monitoring indicators, and payment-triggering conditions
Local governments cannot reach an agreement but aggregate welfare is positive	Participation constraint is not satisfied, but	Upper-level government intervention is needed	Government-led mechanism	Implementation should be supported by fiscal transfers, performance assessment, eco-compensation funds, and environmental inspections
Aggregate welfare is negative	$\Delta W < 0$	The mechanism should not be implemented	Cost sharing should be abandoned	Direct emission control or project-based treatment should be adopted instead

a critical threshold. When $R_{ij} \leq 0.5$, horizontal compensation should not be promoted as the primary instrument. Policymakers should instead rely on standards, enforcement, or upper-level fiscal support. Only when $R_{ij} > 0.5$ does local cost sharing become feasible.

Third, the cost-sharing ratio should be calibrated according to the strength of the externality. The general principle is that the stronger the externality, the higher the appropriate sharing ratio. Under source-region leadership, the sharing ratio tends to increase with the relative externality from the source region to the affected region, and it will only become positive once this externality exceeds a certain threshold. Under affected-region leadership, the sharing ratio also rises with the same externality, but remains more conservative because the affected region internalizes its own payment burden. Under government leadership, the sharing ratios are determined by the relative magnitude of cross-regional externalities in both directions: when the reverse externality from the affected region to the source region is stronger, the corresponding compensation ratio increases, and similarly, when the forward externality from the source region to the affected region is stronger, the associated sharing ratio becomes higher.

In practical policy design, these ratios should be interpreted as calibration principles rather than mechanical formulas. Their application should take into account pollution diffusion, marginal environmental damage, treatment efficiency, and cleanup costs. policy design must distinguish between participation constraints and welfare feasibility. When both are satisfied, local bargaining should be prioritized. When aggregate welfare is positive but the participation constraint is not satisfied, an upper-level government should coordinate through transfer payments or related institutional instruments. When aggregate welfare is negative, the cost-sharing mechanism should not be implemented. leadership structure should be matched with institutional context. Source-region leadership is suitable when the source region controls key treatment resources. However, safeguards such as sharing-ratio ceilings, cost audits, and third-party verification are needed to prevent excessive cost shifting. By contrast, affected-region leadership is more appropriate when the affected region has a strong demand for pollution reduction and can observe source-side treatment performance, whereas government leadership is more appropriate when local bargaining fails or when cross-border externalities require unified monitoring and enforcement.

The affected-region-led mechanism is suitable when the downstream or damaged region has a strong demand for pollution reduction, can observe changes in cross-border pollution, and is willing to subsidize upstream treatment. The advantage of this mechanism is that the compensation motive is clear. However, because the affected region bears the payment cost, the selected compensation ratio is often relatively low and may be insufficient to induce stronger treatment by the source region. In practice, this mechanism can therefore be designed as a performance-based payment scheme, in which compensation is linked to improvements in cross-border water quality, reductions in pollutant concentration, or verified treatment performance, rather than being paid as unconditional reimbursement of source-region expenditure.

The government-led mechanism is suitable when local bargaining fails, when the externality crosses multiple administrative boundaries, when environmental benefits are unevenly distributed, or when unified monitoring and enforcement are required. In the model, the upper-level government should not be understood as an abstract social planner. Rather, it can correspond to provincial governments, river-basin management agencies, central authorities, or cross-regional coordination bodies in real institutional settings. Its role is to internalize cross-border externalities that local governments cannot voluntarily internalize through unified monitoring, target responsibility, fiscal transfers, environmental inspections, performance assessment, and dispute coordination. When voluntary bargaining cannot satisfy participation constraints but cost sharing can improve aggregate welfare, upper-level government intervention becomes particularly important.

China's river-basin and regional governance practices provide institutional examples for this mechanism-selection logic. The upstream–downstream horizontal eco-compensation scheme in the Xin'an River Basin can be understood as a real-world mapping of the beneficiary-pays and source-treatment cost-sharing mechanism in this model. The upper reaches of the Xin'an River are mainly located in Anhui, while downstream water quality is closely related to Zhejiang. Water conservation, pollution reduction, and source-side treatment in the upstream region can directly improve downstream water quality. In terms of the model, this means that source-region treatment generates strong external benefits for the affected region, namely a high R_{ij}. Under this condition, the affected or beneficiary region has an incentive to compensate source-side treatment. Eco-compensation is therefore not merely general fiscal assistance, but a mechanism that increases source-region treatment effort by sharing verifiable cleanup costs.

This case also shows that cost-sharing eco-compensation must be linked to observable and verifiable treatment performance. The institutional core of the Xin'an River compensation mechanism is not simply interregional transfer payment. Rather, compensation is connected with cross-border water-quality monitoring, upstream treatment responsibility, and basin-level environmental performance. In this sense, compensation funds reduce the effective treatment cost of the source region and encourage greater source-side investment. This is consistent with the mechanism identified in this study: compensation does not directly constrain the source region's emission decision, but reduces the marginal cost of source-side treatment, thereby inducing greater cleanup effort and reducing cross-border pollution transmission.

In summary, the policy implication of this study is not merely that eco-compensation should be promoted. Rather, the study proposes a more operational mechanism-selection framework. Policymakers should first identify the direction and strength of the externality, then separately examine participation constraints and aggregate welfare feasibility, then choose between local bargaining and government-led coordination according to the institutional context, and finally ensure implementation through cross-border monitoring, project verification, performance-based payments, and fiscal coordination. Only when the object of compensation, the direction of the externality, the leadership structure, and implementation capacity are mutually aligned can cost-sharing eco-compensation function as an effective incentive mechanism for transboundary pollution control. Otherwise, it may degenerate into general fiscal redistribution and fail to generate substantive environmental improvement.

The study also has several limitations. First, the model considers two regions and therefore does not fully capture multi-region pollution networks. Many real-world watersheds and airsheds involve multiple jurisdictions, nonlinear spillover paths, and networked strategic interactions. Second, the model assumes that cleanup costs and treatment efforts are verifiable and contractible. In practice, eco-compensation schemes may face information asymmetry, monitoring errors, enforcement costs, and political resistance. Third, the functional specifications are stylized to obtain tractable analytical results. Although they capture the key trade-off between production benefits, cleanup costs, and pollution damages, empirical calibration would be needed to evaluate specific policy cases. Future research could extend the framework to multi-region systems, introduce imperfect monitoring and asymmetric information, and examine hybrid instruments that combine cost-sharing eco-compensation with emission taxes, tradable permits, or performance-based environmental contracts.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

YG: Writing – original draft, Formal Analysis, Visualization, Methodology, Conceptualization, Validation. XL: Formal Analysis, Writing – original draft, Conceptualization, Supervision, Writing – review and editing.

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Supplementary material

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fenvs.2026.1857944/full#supplementary-material

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顾宇航同学修读了我主讲的《决策理论与方法》、《采购管理与库存控制》课程，专业基础扎实，理论应用能力较好。

本科期间，顾宇航同学参与了我的研究课题，具备较好的学术潜力和问题分析能力。对待科研认真投入，执行力强。对学术抱有浓厚兴趣，踏实肯钻研。综合其各方面表现，我认为该生满足推免硕士研究生的相关条件，特此予以推荐。

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顾宇航同学大一时修读了我讲授的新生研讨课《智慧物联与智慧医疗》课程，该同学学习态度积极认真，课堂学习投入，能够较好地掌握相关理论知识，积极参加课堂研讨和课后小组研讨，成绩优秀。

该同学积极参加国内外学术交流活动，善于思考，治学严谨，具有良好的创新意识和科研潜力，以第一作者身份发表一篇SCI期刊论文，展现出较强的学术素养和科研能力。

综上，该同学成绩优秀，科研能力突出，同意推荐。

推荐人签名 [signature]

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